Concurrent Trading in Two Experimental Markets with Demand Interdependence

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Summary. We report results from fifteen computerized double auctions with concurrent trading of two commodities. In contrast to prior experimental markets, buyers' demands are induced via CES earnings functions defined over the two traded goods, with a fiat money expenditure constraint. Sellers receive independent marginal cost arrays for each commodity. Parameters for buyers' earnings functions and sellers' costs are set to yield a stable, competitive equilibrium. In spite of the complexity introduced by the demand interdependence, the competitive model is a good predictor of market outcomes, although prices tend to be above (below) the competitive prediction in the low-price (high-price) market.

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1 Introduction

One of the most widely known and well documented results from laboratory experimentation with private good allocation mechanisms is that double auction (DA) trading of a single commodity under conditions of a static competitive equilibrium (CE) will generate actual price and quantity observations that are near the CE.[1] This CE convergence property is quite robust with respect to the number of market participants, the specific supply and demand configurations used (for example, see Smith and Williams [1990]). DA market performance has rarely been surpassed by alternative institutions to which it has been systematically compared.[2]

This paper reports fifteen experimental sessions examining the predictive power of the competitive model in a two-market trading environment with demand interdependence. The demand implementation method corresponds to an integer-discrete version of the standard two-good indifference curve exposition of consumer theory presented in classrooms as the theoretical foundation for the demand functions utilized in competitive market analysis. Trading decisions are made in real time in the presence of considerable price uncertainty as the market adjusts toward a behavioral equilibrium. Several initialization parameters are systematically varied across experimental replications in order to explore the sensitivity of market outcomes to variations in market structure within the basic two-commodity trading framework described below.

Consider a laboratory market with m buyers and n sellers trading two pure private goods, A and B, whose fiat money denominated prices P_A and P_B are individually negotiated

[1] This conclusion is for classical environments, but exceptions were reported by Smith [1965] where 11 buyers each have one unit valued at \$4.60 and 13 sellers each have one unit costing \$3.10. With constant excess supply of only 2 units, convergence was incomplete and erratic. A similar result has been demonstrated by Holt, Langan, and Villamil [1986] and by Davis and Williams [1991], in the context of a "market power" design where five buyers and five sellers each have five units, and two sellers need only withhold one unit each to convert one unit of excess supply to one unit of excess demand. In non-classical environments with externalities or indivisibilities, the double auction fails to yield efficient competitive equilibria. For externalities, see Plott [1983]; for an example of indivisibilities in which firms have avoidable fixed costs, see Van Boening and Wilcox [1995] and Durham et al. [1995]. With indivisibilities, the market failure occurs because of the tendency of the double auction to converge to one price. Smith and Williams [1992] provide a nontechnical comparison of computerized DA, posted offer, and call market institutions with a nonstationary CE.

[2] Modest exceptions include one of four different continuous feedback uniform-price auctions (McCabe, Rassenti, and Smith [1991]), and the double-dutch auction (McCabe, Rassenti, and Smith [1992]).

for each unit traded. Agents are assumed to have monotonic increasing subjective utility functions U_i for U.S. currency. Each buyer $i \in \{1, 2, \ldots, m\}$ is given (1) an experimenter induced quasi-concave earnings function $\pi_i(A_i, B_i)$ which associates a specific cash payment with integer commodity bundles[3] and (2) an endowment of T_i "tokens" that can be used to purchase units of either good. One can think of T_i as implicitly being generated by a labor endowment, L_i, supplied to sellers at a token-wage of w such that $T_i = w \cdot L_i$. Buyer i can continue to purchase commodity units as long as the price is less than or equal to the number of "tokens remaining" defined as T_i minus expenditures on goods A and B.

A utility maximizing buyer would be concerned with maximizing $U_i[\pi_i(A_i, B_i)]$ subject to the budget constraint implied above. Assuming P_A and P_B constant, the solution set of the above constrained maximization problem yields m individual demand functions for each commodity and thus market demand functions $D_A(P_A, P_B)$ and $D_B(P_B, P_A)$.

Each seller has an additively separable induced token-cost function $C_i(A_i, B_i)$ presented in the form of increasing, independent marginal cost arrays for integer increments of each good. Sellers earn the difference between price and marginal cost for each unit traded plus a 0.05 token commission to cover subjective transaction costs. Sellers' token profits are automatically converted into U.S. dollars on a one to one basis. The sellers' cost functions can be thought of as implicitly derived from independent labor-product transformation functions and a token-wage rate of w paid to buyers for each labor input unit supplied.

Assuming P_A and P_B constant, the marginal cost arrays which correspond to n individual supply arrays thus determine the aggregate market supply functions $S_A(P_A)$ and $S_B(P_B)$. The individual demand and cost parameters are such that a unique Walrasian equilibrium exists in each market, with equilibrium prices (P_A^*, P_B^*) and quantities (Q_A^*, Q_B^*) determined by the equations $D_A(P_A^*, P_B^*) = S_A(P_A^*) = Q_A^*$ and $D_B(P_B^*, P_A^*) = S_B(P_B^*) = Q_B^*$, where P_A and P_B are measured in terms of tokens (or labor), the third commodity.[4] The general equilibrium closure conditions are thus satisfied at (P_A^*, P_B^*): (1) all units of goods A and B produced by sellers are consumed by buyers, and (2) all token (labor) endowments to buyers

[3]See Smith [1982] for a more formal and comprehensive discussion of induced valuation techniques in experimental environments.

[4]In future studies we plan to examine markets with multiple and unstable equilibria. For an experimental design of exchange economies with multiple equilibria, see Gjerstad [1996]. The first step obviously is to analyze the behavior of theoretically stable markets as an empirical base from which we can move to explore successively more complex markets. Also, multiple commodity environments are well suited to experimental examinations of entry-exit decisions such as those addressed by Isaac and Smith [1985] and Coursey, Isaac, and Smith [1984].

are paid out to sellers.

Since our method of inducing (interdependent) utility on two commodities is a substantial change relative to the use of explicitly stated marginal valuations, it is an open question as to whether, or in what sense, the results of one-commodity experiments will extend to this new environment.[5] In one-commodity markets, buyer i's maximum willingness-to-pay for each successive unit is well-defined by the "resale values" (limit prices) assigned to i for successive units that might be purchased. This follows if we assume merely that buyers are nonsatiated in money since buyer i is paid in cash the difference between the assigned value of each unit purchased and the unit's purchase price (realized consumer surplus) in the experimental market. In the two-commodity market just described, maximum willingness-to-pay for each successive unit of A is defined by the uncertain opportunity costs of spending tokens in market B and vice versa. This opportunity cost demand, relative to limit price demand, may be so weak that the convergence properties of one-commodity markets fail to extend to this new environment.

This paper is organized as follows: Section 2 explains the two-commodity double-auction trading mechanism. Section 3 explains the experimental design and initialization of market parameters. Section 4 reports the results of the experimental markets and is divided into three subsections, one focusing on market price and quantity convergence to the CE, one focusing on individual buyer quantity decisions, and the last is a brief summary of our experimental results.

2 Two commodity double auction trading

The market software employed in this study is a revised version of the one-commodity DA mechanism developed by Smith and Williams (Williams [1980], Smith and Williams [1983]). Figure 1 (2) shows the basic screen display for a buyer (seller) during a market session.

Bargaining and exchange in both the market for good A and the market for good B occurs simultaneously. Buyers and sellers indicate the market in which they desire to be currently

[5] Forsythe, Palfrey, and Plott [1984] use an earnings function that depends on individual holdings of an asset at the end of two distinct trading periods in a double-auction market. While their study does deviate from the traditional (unit independent) induced valuation structure, the specific focus of their work is very different from ours. Our interdependent reward structure is somewhat akin to Smith's [1980] experiments with a public good allocation mechanism using a Cobb-Douglas reward function defined over final holdings of a private and a public good.

active by touching the rectangular area on their display screen labeled "MARKET A" or "MARKET B". The area corresponding to the market in which the subject is currently active is shaded as a reminder of this choice. Market participants are able to switch markets at any time during a trading period.

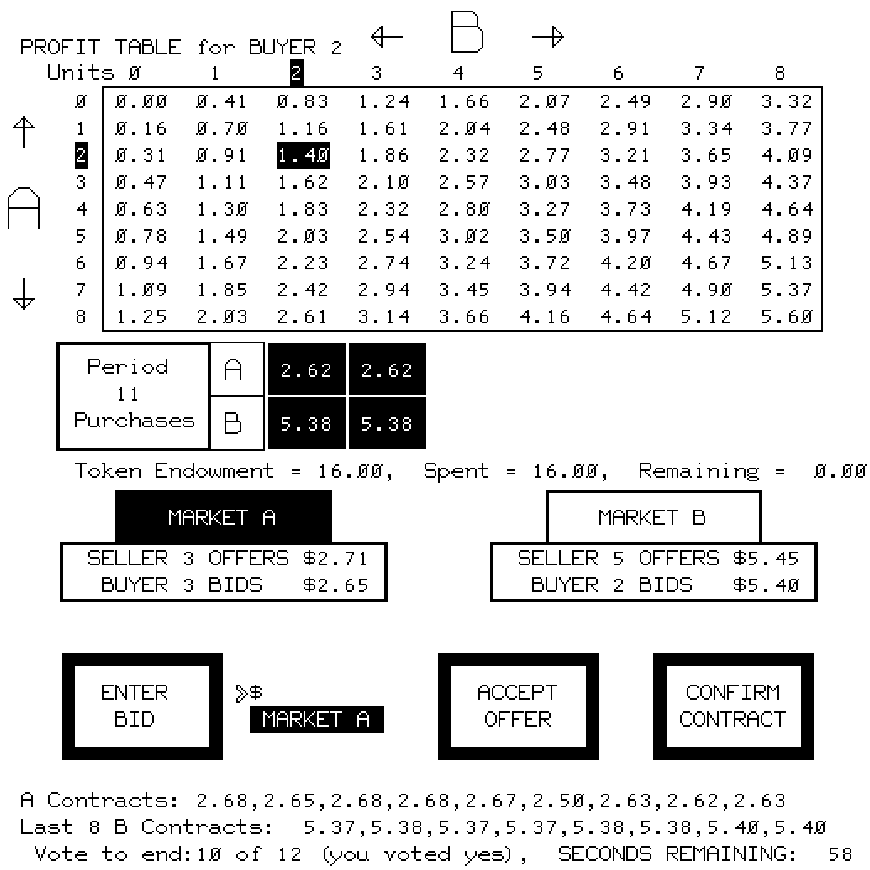


Figure 1: Buyer's screen display in two-commodity trading experiment.

Buyers (sellers) enter bids to buy (offers to sell) one commodity unit by typing their entry and then touching the rectangular area on their display screen labeled "ENTER BID" ("ENTER OFFER") at which time the entry is made public unless it violates a DA procedural rule. Any buyer (seller) is free to accept any seller's offer (buyer's bid) in the market in which he/she is currently active by touching a display screen area labeled "ACCEPT OFFER" ("ACCEPT BID"). The acceptor must then touch an area labeled "CONFIRM CONTRACT" at which time a binding contract is formed and the information is logged in both the maker's and taker's private record sheets. Buyers are not allowed to enter bids or

RECORD SHEET for SELLER 4 TRADING PERIOD 11

Commodity A	Units→ 1	2	3	4	5
Selling Price	2.68	2.67			
Production Cost	2.20	2.50	2.80	3.10	3.40
Profit + $.05 commission	0.53	0.22			

Commodity B	Units→ 1	2	3	4	5
Selling Price	5.37	5.40			
Production Cost	4.90	5.40	5.50	6.00	6.10
Profit + $.05 commission	0.52	0.05			

MARKET A
SELLER 3 OFFERS $2.71
BUYER 3 BIDS $2.65

MARKET B
SELLER 5 OFFERS $5.45
BUYER 2 BIDS $5.40

ENTER OFFER   >$ MARKET A   ACCEPT BID   CONFIRM CONTRACT

A Contracts: 2.68,2.65,2.68,2.68,2.67,2.50,2.63,2.62,2.63
Last 8 B Contracts: 5.37,5.38,5.37,5.37,5.38,5.38,5.40,5.40
Vote to end:10 of 12 (you voted yes), SECONDS REMAINING: 57

Figure 2: Seller's screen display in two-commodity trading experiment.

accept offers that exceed their "tokens remaining". Sellers are not allowed to enter offers or accept bids that are below the marginal cost of the unit being sold. Price quotes which violate these rules automatically generate a descriptive error message and are subsequently ignored by the auction system.

Price quotes must progress so as to reduce the bid-ask spread. Only the highest bid to buy and the lowest offer to sell in each market are displayed to the participants and are open to acceptance. Any quotation which does not provide better terms to the other side of the market is placed in a queue which ranks bids from highest to lowest (offers from lowest to highest). After a contract occurs, the highest queued bid and the lowest queued offer are automatically entered as the new bid-ask spread. The price quotes contained in each queue are unknown to market participants, however, the maker of a queued price quote is given continuously updated information on the position of the quote in the queue. Queued entries may be withdrawn at any time by pressing a key labeled -EDIT-. If queued, participants must exit the queue before being allowed to switch markets. Note that the "rank-queue" in each market is simply an electronic limit order file or "specialist's book". Smith and Williams [1983] found that this element of the DA trading mechanism significantly improved (relative to three alternatives) the rate of convergence to the CE in single commodity DA markets.

It is important to stress that bids, offers, and subsequent contracts in both markets are the only public price information made available to market participants through the DA mechanism. The bid-ask spread and a list of recent contract prices for both markets is presented to each subject regardless of the market in which they are active.

Trading takes place during a sequence of market periods each lasting a maximum of 360 seconds. Buyers' token endowments are replenished prior to the beginning of each trading period and buyers begin each period at the $(A_i, B_i) = (0, 0)$ commodity bundle. Unspent tokens can not be carried over from one trading period to another. Participants can bypass the 360 second stopping rule by unanimously voting to end the trading period. Registering a vote to end a period does not affect the individual's ability to actively participate in the markets. The bid-ask spread, contract prices, the number of seconds remaining, and the current vote to end the period are presented as shown at the bottom of Figures 1 and 2. Screen displays are updated every one or two seconds. At the end of each trading period, participants are able to review all contract prices and their personal record sheets from any completed trading period.

3 Experimental design

In all experiments reported below, the profit function of buyer i has a constant elasticity of substitution (CES) specification $\pi_i(A, B) = \delta_i \cdot (\alpha_i A^{\rho_i} + \beta_i B^{\rho_i})^{1/\rho_i}$. The ρ_i parameter determines the buyer's elasticity of commodity substitution, $\frac{1}{1-\rho_i}$, and hence the curvature of the buyer's indifference contours. The function reduces to Cobb-Douglas as $\rho \to 0$, has linear indifference contours for $\rho = 1$, and tends to Leontief preferences as $\rho \to -\infty$. The CES parameter values $\{(\delta_i, \alpha_i, \beta_i, \rho_i)\}_{i=1}^m$ and the token endowments $\{T_i\}_{i=1}^m$ are initialized by first specifying the desired CE price (P_A^*, P_B^*) in each market, then specifying (for each buyer) integer CE quantities of both commodities, the profit associated with the CE bundle, and ρ_i. Using the first-order conditions for utility maximization, the computer then calculates and stores each buyer's token endowment T_i, α_i, and $\beta_i = (1-\alpha_i)$. The δ_i parameter is then set so as to yield the desired profit at the CE bundle. Each seller's marginal cost arrays are then initialized by the experimenter to generate market supply arrays that are consistent with the CE prices and aggregate quantities specified in the demand initialization procedure.

6 Buyer Market

	Parameter				
Buyer	ρ	δ	α	β	T
1	0.25	0.554	0.174	0.826	23.50
2	0.75	0.580	0.268	0.732	23.50
3	0.25	0.700	0.325	0.675	20.00
4	0.75	0.700	0.325	0.675	20.00
5	0.25	0.759	0.523	0.477	16.50
6	0.75	0.818	0.388	0.612	16.50

4 Buyer Market

	Parameter				
Buyer	ρ	δ	α	β	T
1	0.25	0.606	0.223	0.777	33.50
2	0.75	0.781	0.364	0.636	26.50
3	0.25	0.759	0.447	0.553	26.50
4	0.75	0.620	0.288	0.712	33.50

Table 1: Buyer parameters in 6 buyer and 4 buyer base markets.

	Units of A					Units of B				
Seller	1	2	3	4	5	1	2	3	4	5
1	2.95	3.05	3.55	3.65	4.15	6.35	6.65	6.95	7.25	7.55
2	2.75	3.25	3.35	3.85	3.95	6.25	6.75	6.85	7.35	7.45
3	2.75	3.25	3.35	3.85	3.95	6.45	6.55	7.05	7.15	7.65
4	2.85	3.15	3.45	3.75	4.05	6.25	6.75	6.85	7.35	7.45
5	2.95	3.05	3.55	3.65	4.15	6.35	6.65	6.95	7.25	7.55
6	2.85	3.15	3.45	3.75	4.05	6.45	6.55	7.05	7.15	7.65

Table 2: Sellers' unit marginal costs for goods A and B.

The specific buyer parameters used in our base market designs are given in Table 1. Sellers' unit marginal cost schedules for the base market design are given in Table 2. The markets have either four or six buyers and six sellers who, at a CE, exchange a total of twelve units in each market. Figure 3 displays buyer profit tables generated by the parameter values shown in Table 1 for a six-buyer market. Figure 4 displays buyer profit tables for a four-buyer market. The highlighted quantities and payoffs in these tables are the equilibrium quantities and payoffs for each buyer. At the CE, the aggregate earnings of buyers is \$8.40 per trading period in both the four-buyer and six-buyer designs while sellers earn \$3.00 per period in each market, net of commissions (\$7.20 total including commissions).

PROFIT TABLE for BUYER 1 ← B →

Units	0	1	2	3	4	5	6	7	8
0	0.00	0.26	0.51	0.77	1.03	1.29	1.54	1.80	2.06
1	0.00	0.55	0.99	1.40	1.80	2.18	2.56	2.94	3.30
2	0.00	0.63	1.11	1.55	1.98	2.39	2.80	3.20	3.59
3	0.00	0.69	1.19	1.66	2.11	2.54	2.97	3.39	3.80
4	0.00	0.73	1.26	1.75	2.22	2.67	3.10	3.54	3.96
5	0.00	0.77	1.32	1.82	2.31	2.77	3.22	3.66	4.10
6	0.00	0.81	1.37	1.89	2.38	2.86	3.32	3.78	4.22
7	0.00	0.84	1.42	1.95	2.46	2.94	3.41	3.88	4.33
8	0.00	0.87	1.46	2.01	2.52	3.02	3.50	3.97	4.43

PROFIT TABLE for BUYER 2 ← B →

Units	0	1	2	3	4	5	6	7	8
0	0.00	0.38	0.77	1.15	1.53	1.91	2.30	2.68	3.06
1	0.10	0.58	0.99	1.40	1.80	2.20	2.59	2.99	3.38
2	0.20	0.73	1.16	1.58	1.99	2.40	2.80	3.20	3.60
3	0.30	0.86	1.31	1.74	2.16	2.57	2.98	3.39	3.80
4	0.40	0.99	1.45	1.89	2.32	2.74	3.16	3.57	3.98
5	0.50	1.11	1.59	2.04	2.47	2.90	3.32	3.74	4.15
6	0.60	1.23	1.72	2.18	2.62	3.05	3.48	3.90	4.32
7	0.70	1.35	1.85	2.31	2.76	3.20	3.63	4.06	4.48
8	0.80	1.47	1.97	2.45	2.90	3.35	3.78	4.21	4.64

PROFIT TABLE for BUYER 3 ← B →

Units	0	1	2	3	4	5	6	7	8
0	0.00	0.15	0.29	0.44	0.58	0.73	0.87	1.02	1.16
1	0.01	0.70	1.13	1.52	1.88	2.22	2.55	2.87	3.18
2	0.02	0.89	1.40	1.85	2.26	2.66	3.03	3.40	3.75
3	0.02	1.04	1.60	2.10	2.56	2.99	3.40	3.79	4.18
4	0.03	1.16	1.78	2.31	2.80	3.26	3.70	4.12	4.53
5	0.04	1.27	1.93	2.50	3.01	3.50	3.96	4.41	4.84
6	0.05	1.37	2.07	2.67	3.21	3.72	4.20	4.66	5.11
7	0.05	1.47	2.20	2.82	3.39	3.92	4.42	4.90	5.36
8	0.06	1.56	2.32	2.97	3.55	4.10	4.62	5.12	5.60

PROFIT TABLE for BUYER 4 ← B →

Units	0	1	2	3	4	5	6	7	8
0	0.00	0.41	0.83	1.24	1.66	2.07	2.49	2.90	3.32
1	0.16	0.70	1.16	1.61	2.04	2.48	2.91	3.34	3.77
2	0.31	0.91	1.40	1.86	2.32	2.77	3.21	3.65	4.09
3	0.47	1.11	1.62	2.10	2.57	3.03	3.48	3.93	4.37
4	0.63	1.30	1.83	2.32	2.80	3.27	3.73	4.19	4.64
5	0.78	1.49	2.03	2.54	3.02	3.50	3.97	4.43	4.89
6	0.94	1.67	2.23	2.74	3.24	3.72	4.20	4.67	5.13
7	1.09	1.85	2.42	2.94	3.45	3.94	4.42	4.90	5.37
8	1.25	2.03	2.61	3.14	3.66	4.16	4.64	5.12	5.60

PROFIT TABLE for BUYER 5 ← B →

Units	0	1	2	3	4	5	6	7	8
0	0.00	0.04	0.08	0.12	0.16	0.20	0.24	0.27	0.31
1	0.06	0.76	1.07	1.33	1.56	1.77	1.97	2.16	2.34
2	0.11	1.11	1.52	1.85	2.14	2.41	2.66	2.90	3.12
3	0.17	1.40	1.89	2.28	2.62	2.93	3.22	3.49	3.75
4	0.23	1.66	2.21	2.65	3.04	3.38	3.70	4.00	4.29
5	0.28	1.91	2.52	3.00	3.42	3.79	4.14	4.47	4.78
6	0.34	2.14	2.80	3.32	3.77	4.18	4.55	4.90	5.24
7	0.40	2.36	3.07	3.63	4.11	4.54	4.94	5.31	5.67
8	0.46	2.57	3.33	3.92	4.43	4.89	5.31	5.70	6.07

PROFIT TABLE for BUYER 6 ← B →

Units	0	1	2	3	4	5	6	7	8
0	0.00	0.43	0.85	1.28	1.70	2.13	2.55	2.98	3.40
1	0.23	0.82	1.30	1.77	2.23	2.68	3.13	3.57	4.02
2	0.46	1.12	1.64	2.13	2.60	3.07	3.54	4.00	4.45
3	0.69	1.40	1.94	2.45	2.95	3.43	3.91	4.38	4.84
4	0.93	1.67	2.24	2.76	3.27	3.77	4.25	4.73	5.21
5	1.16	1.94	2.52	3.06	3.58	4.09	4.59	5.08	5.56
6	1.39	2.20	2.80	3.36	3.89	4.40	4.91	5.40	5.90
7	1.62	2.46	3.07	3.64	4.18	4.71	5.22	5.73	6.22
8	1.85	2.71	3.34	3.92	4.47	5.01	5.53	6.04	6.54

Figure 3: Buyers' profit tables: six-buyer markets.

In one-commodity market experiments it is quite common to shift the individual induced marginal valuation and cost arrays (and thus the CE) by an arbitrary, uniform additive constant in order to disguise the CE price when replicating experiments using the same subject population. The technique employed to accomplish this in a two-commodity environment with interdependent demand is not quite as simple as in the one-commodity case. In order to implement a rescaling that is (potentially) behaviorally benign, the ratio of market prices must be held constant. However, this means that the CE price gap between markets will be altered. When replicating the basic market designs shown in Table 1, we varied: (1) the designation of A or B as the market with the higher (lower) CE price, and (2) the absolute size of the CE price gap ($|P_A^* - P_B^*| = 2.80$, 3.50, or 4.20), holding the ratio of CE prices constant at 2.077. The latter was accomplished by multiplying the token endowments and CE prices (P_A^*, P_B^*) shown in Table 1 by either 0.8 or 1.2 and shifting the supply arrays by an additive constant such that $S_A(P_A^*) = S_B(P_B^*) = 12$. The effects of this rescaling on

PROFIT TABLE for BUYER 1 B

A Units	0	1	2	3	4	5	6	7	8
0	0.00	0.22	0.44	0.66	0.89	1.11	1.33	1.55	1.77
1	0.00	0.61	1.05	1.46	1.85	2.23	2.60	2.97	3.32
2	0.00	0.72	1.21	1.67	2.10	2.52	2.92	3.32	3.70
3	0.00	0.80	1.33	1.82	2.28	2.72	3.15	3.57	3.98
4	0.01	0.86	1.43	1.94	2.43	2.89	3.34	3.77	4.20
5	0.01	0.92	1.52	2.05	2.55	3.03	3.50	3.95	4.39
6	0.01	0.97	1.59	2.15	2.66	3.16	3.64	4.10	4.56
7	0.01	1.02	1.66	2.23	2.77	3.28	3.77	4.24	4.71
8	0.01	1.07	1.73	2.31	2.86	3.38	3.89	4.37	4.85

PROFIT TABLE for BUYER 2 B

A Units	0	1	2	3	4	5	6	7	8
0	0.00	0.43	0.85	1.28	1.71	2.14	2.56	2.99	3.42
1	0.20	0.78	1.26	1.73	2.19	2.64	3.09	3.53	3.98
2	0.41	1.05	1.56	2.05	2.53	2.99	3.46	3.91	4.37
3	0.61	1.30	1.84	2.34	2.83	3.31	3.79	4.26	4.72
4	0.81	1.54	2.10	2.62	3.13	3.62	4.10	4.58	5.05
5	1.02	1.78	2.35	2.89	3.40	3.91	4.40	4.89	5.37
6	1.22	2.01	2.60	3.15	3.68	4.19	4.69	5.18	5.67
7	1.42	2.24	2.85	3.41	3.94	4.46	4.97	5.47	5.96
8	1.63	2.47	3.09	3.66	4.20	4.73	5.24	5.75	6.25

PROFIT TABLE for BUYER 3 B

A Units	0	1	2	3	4	5	6	7	8
0	0.00	0.07	0.14	0.21	0.28	0.35	0.42	0.50	0.57
1	0.03	0.76	1.13	1.44	1.73	2.00	2.25	2.49	2.73
2	0.06	1.05	1.52	1.91	2.26	2.58	2.89	3.18	3.46
3	0.09	1.29	1.83	2.28	2.67	3.04	3.39	3.72	4.03
4	0.12	1.50	2.10	2.60	3.03	3.44	3.82	4.17	4.52
5	0.15	1.69	2.35	2.88	3.36	3.79	4.20	4.58	4.95
6	0.18	1.87	2.58	3.15	3.66	4.12	4.55	4.96	5.35
7	0.21	2.04	2.79	3.40	3.94	4.42	4.88	5.31	5.72
8	0.24	2.20	3.00	3.64	4.20	4.71	5.19	5.64	6.07

PROFIT TABLE for BUYER 4 B

A Units	0	1	2	3	4	5	6	7	8
0	0.00	0.39	0.79	1.18	1.58	1.97	2.36	2.76	3.15
1	0.12	0.62	1.05	1.47	1.88	2.29	2.70	3.11	3.51
2	0.24	0.79	1.24	1.67	2.10	2.52	2.94	3.35	3.77
3	0.35	0.94	1.41	1.86	2.30	2.73	3.15	3.57	3.99
4	0.47	1.09	1.57	2.03	2.48	2.92	3.35	3.78	4.20
5	0.59	1.23	1.73	2.20	2.65	3.10	3.54	3.97	4.40
6	0.71	1.37	1.88	2.36	2.82	3.27	3.72	4.16	4.59
7	0.83	1.51	2.03	2.52	2.99	3.44	3.89	4.34	4.78
8	0.94	1.65	2.18	2.67	3.15	3.61	4.07	4.51	4.96

Figure 4: Buyers' profit tables: four-buyer markets.

price dynamics are, a priori, unclear but our working hypothesis is that dynamics will be unaffected by the CE price gap differential. In Section 4 we report some evidence regarding the empirical validity of this hypothesis.

Table 3 displays the value of P_A^* and P_B^*, the number of buyers and sellers, and the final trading period for each experiment. The final period was set so that each experimental session lasted between 2 and 2.5 hours. All subjects were experienced with the basic trading mechanism in the sense that each had participated in at least one single-commodity computerized DA experiment.[6] Experiments with an "x" suffix used subjects who had also participated in a previous two-commodity experiment (level 2 experience). Buyers in level 2 experiments were also buyers in their first two-commodity experiment. In the other nine experiments subjects had no previous experience with the two-commodity environment (level 1 experience). All subjects were volunteers drawn from the student populations at Indiana University and the University of Arizona. Most of the experiments were run "multisite" with subjects participating simultaneously from labs at both locations.

After arriving at the experiment site, participants were each paid $3 for keeping their appointment and were then randomly assigned to individual computer terminals. The double-auction program then (1) assigns each terminal to the buyer or seller condition, (2) presents

[6]One pilot experiment using completely inexperienced subjects was run. It was clear that the two-commodity environment was too complex for subjects with no previous exposure to computerized DA markets.

Experiment	Number of buyers	Number of sellers	P_A^*	P_B^*	Final Period
4pda06	6	6	3.25	6.75	11
4pda07	6	6	6.75	3.25	11
4pda10	6	6	2.60	5.40	13
4pda11	6	6	8.10	3.90	13
4pda14	6	6	5.40	2.60	11
4pda16	6	6	3.90	8.10	10
4pda12	4	6	8.10	3.90	12
4pda17	4	6	2.60	5.40	10
4pda19	4	6	3.25	6.75	10
4pda08x	6	6	2.60	5.40	11
4pda18x	6	6	6.75	3.25	11
4pda20x	6	6	8.10	3.90	11
4pda09x	4	6	8.10	3.90	12
4pda13x	4	6	2.60	5.40	11
4pda15x	4	6	3.25	6.75	13

Table 3: Experiment classification.

the instructions at an individually controlled pace, (3) waits for everyone to finish the instructions, and then (4) executes the experiment and stores the resulting data on disk for later recall and analysis.

4 Experimental results

Market convergence to CE Figure 5 displays 9 sets of time series for each of the experience level 1 markets. The graph shows, for each of these 9 markets, the average price in each period in the high-price and in the low-price market. The mean contract prices for the experience level 2 markets are shown in Figure 6.[7] The time series of individual contract

[7]Ten (of more than 3500 total) price observations are deleted as outliers. These contract prices represent documented human errors by buyers who made a contract in the low-price market when they thought they were contracting in the high-price market. The erroneous contracts occurred in: 4pda07, period 3; 4pda09x, periods 2, 4, 9; 4pda10, period 3; 4pda11, period 8 (twice by the same buyer); 4pda15x, period 13; 4pda17, period 9; 4pda20, period 3. The markets showed no sustained price reaction to the errant contracts. In all cases the price series returned to their previous range immediately; it appeared that everyone was aware that

prices for three selected experiments are presented in Figures 7 – 9. In these figures, each dot represents a transaction price. Transactions in successive periods are separated by a vertical line, and the number of transactions in each period is indicated below the vertical lines separating periods. It is clear from these charts that the CE has considerable predictive power although CE convergence is not as rapid nor as pervasive as in single-commodity DA markets with a stationary CE and similarly experienced traders. Given the task complexity inherent in our two-commodity environment, this result is not terribly surprising.

The price data presented in Figures 5 – 9 are summarized in Figure 10 by the 95% confidence bands for the time sequence of population period-mean prices. Figure 10 shows that prices tend to stabilize slightly below the CE in the high-price markets and slightly above the CE in the low-price markets. (Two experiments, 4pda09 and 4pda17, clearly run counter to this general result.) After about five trading periods, the confidence bands are quite tight and nearly stationary. This behavioral price equilibrium is further characterized in Figure 11 which displays histograms of individual contract prices in the low-price and in the high-price markets pooled across all replications of periods 6 - 10. Although prices tend to be "close" to the CE in both sample distributions, t-tests indicate that we should reject the null hypothesis that the population mean is equal to the CE prediction ($t = -11.3$ for the high-price market and $t = 5.7$ for the low-price market).[8]

In addition to the generally small, although sustained, price deviations from the CE, Figures 7 – 9 reveal a tendency for market volume to be slightly higher in the low-price markets than in the high price markets (even though the CE volume is 12 in both). This is also reflected in Figure 11 by the difference in the sample sizes for the high-price and low-price frequency polygons. Over periods 6 - 10, trading volume in low-price markets is 98.4% of the CE prediction (889 of 900) while in high-price markets trading volume is 90% of the CE prediction (810 of 900). Further, the mean volume in the low-price markets is larger than in the high-price markets in all trading periods.

the contract simply represented a human error and thus did not significantly affect anyone's price or profit expectations. It should also be noted that similar errors were impossible in the high-price market since sellers were not able to enter or accept price quotes below marginal cost (just as buyers were not able to enter or accept quotes that violated their budget constraints).

[8] These sample distributions use all price observations (except as noted in footnote 5) rather than the fifteen period-mean prices. While this grand pooling captures intraperiod price variation it also is likely to violate the independence assumption needed to apply the t-distribution in the hypothesis test. This t-test should thus be viewed as describing an average price deviation from CE price rather than as a formal statistical result.

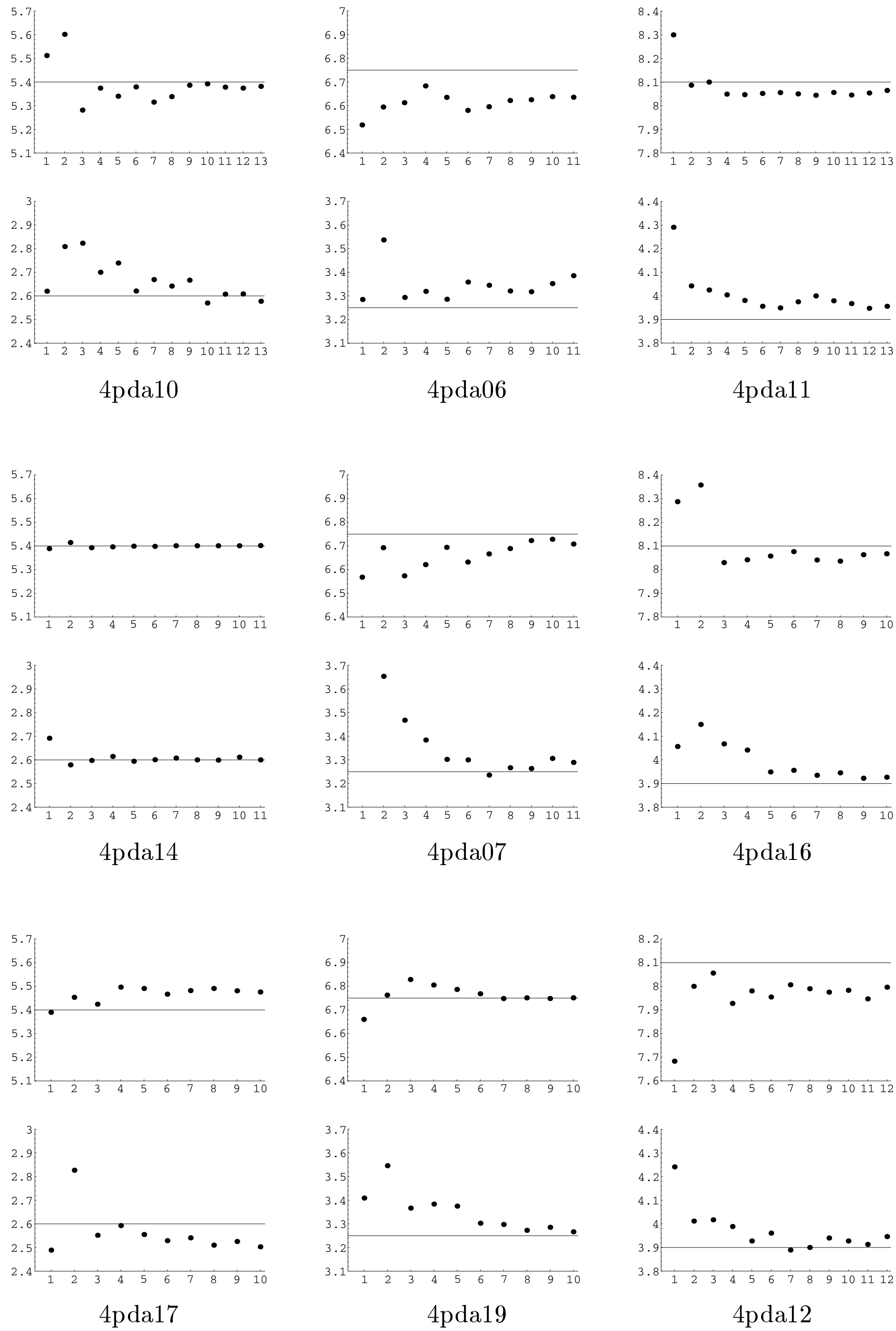


Figure 5: Mean contract prices by period: experience level 1 markets.

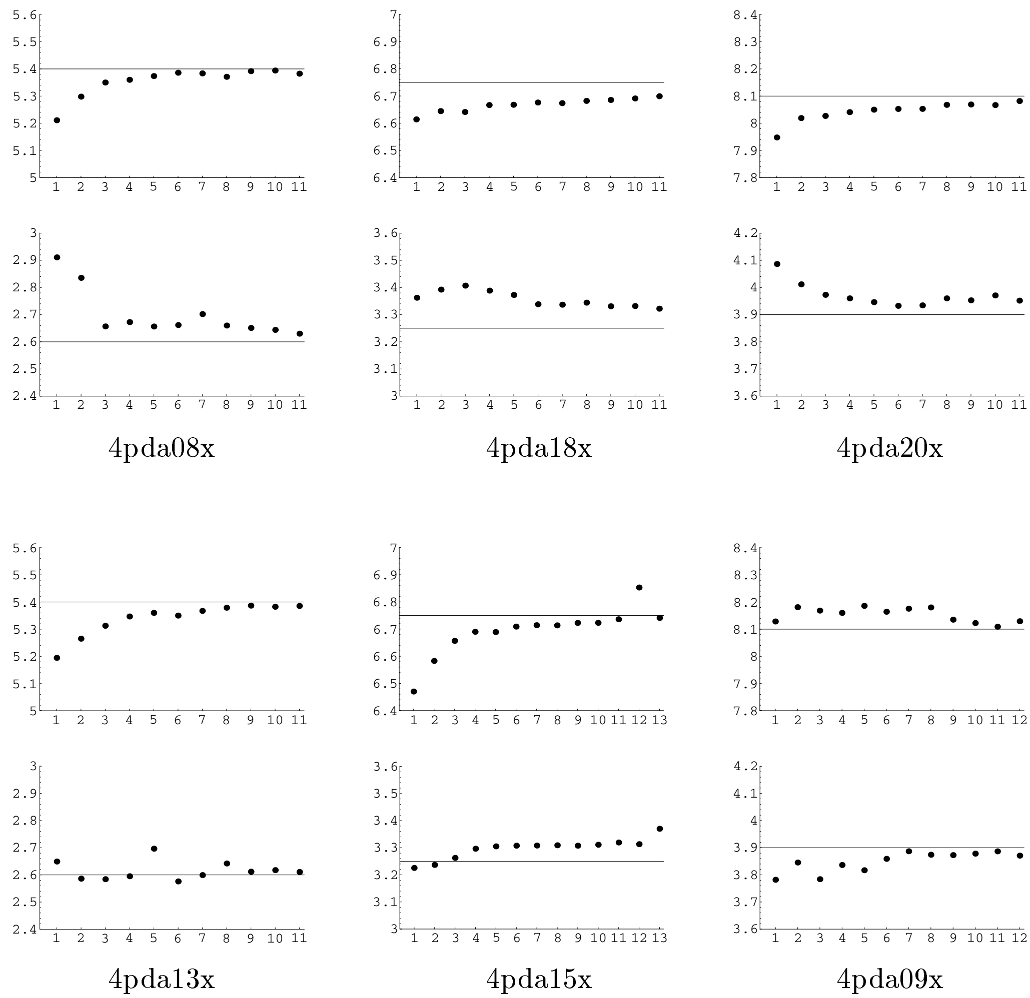


Figure 6: Mean contract prices by period: experience level 2 markets.

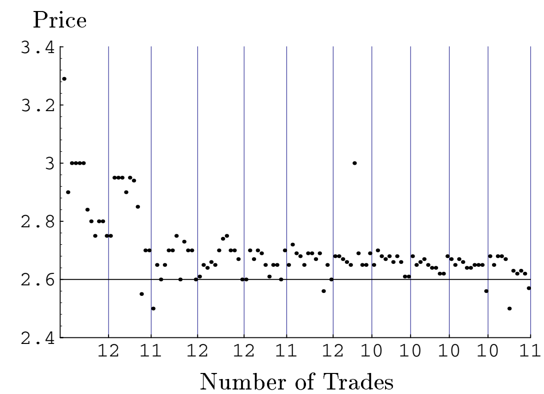
The marginal (linear) effect of the market design variations on the speed of price converge to the CE is summarized by OLS estimation of the following equation:

$$r(t) = a + b\,t + c\,X + d\,H + e\,L + f\,F + g\,B + u_t.$$

The dependent variable, $r(t)$, is the root-mean-square-error of contract prices from the CE price prediction in period t:

$$r(t)^2 = \frac{1}{Q(t)} \cdot \sum_{i=1}^{Q(t)} [P_i(t) - CE]^2,$$

where $P_i(t)$ and $Q(t)$ are the i^{th} contract price and the number of contracts in trading period t, respectively. Since $r(t)$ depends on both the mean and variance of prices, $r(t) = 0$ only



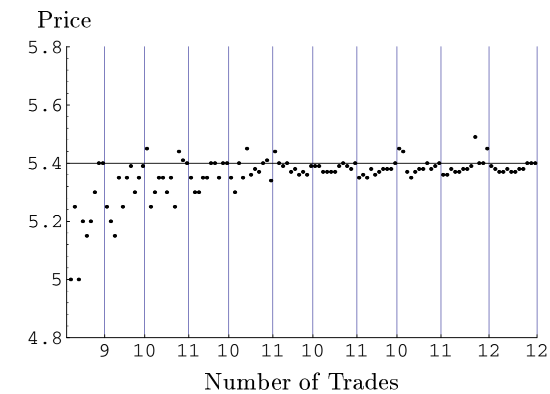
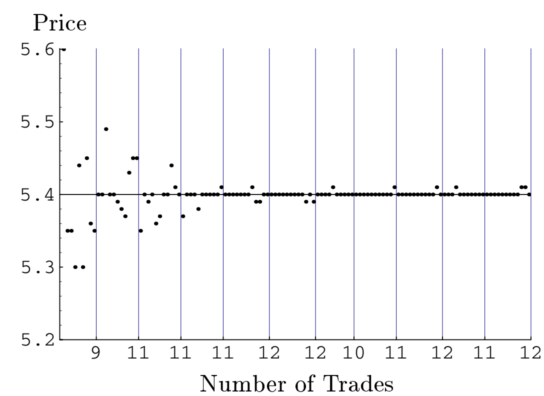


Figure 7: Prices in markets A (left) and B (right) for experiment 4pda08.



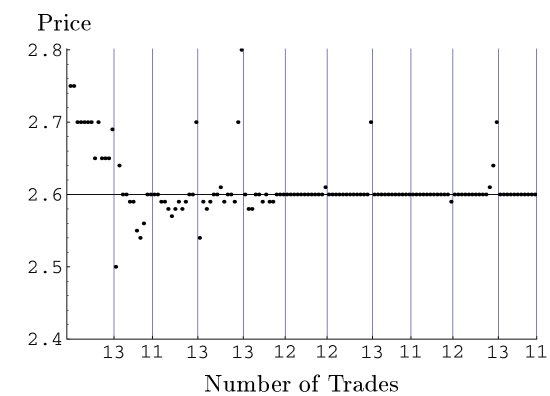
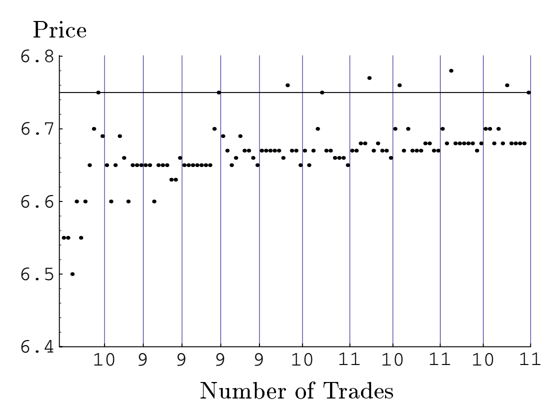


Figure 8: Prices in markets A (left) and B (right) for experiment 4pda14.



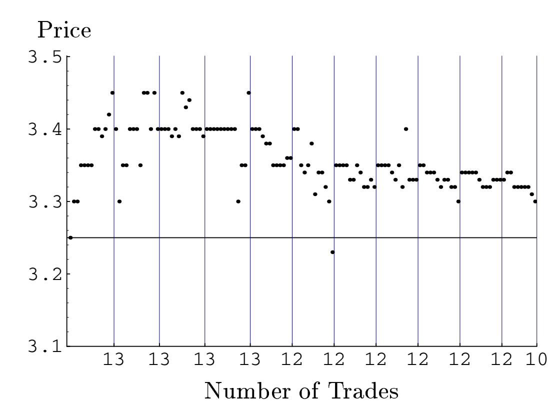


Figure 9: Prices in markets A (left) and B (right) for experiment 4pda18.

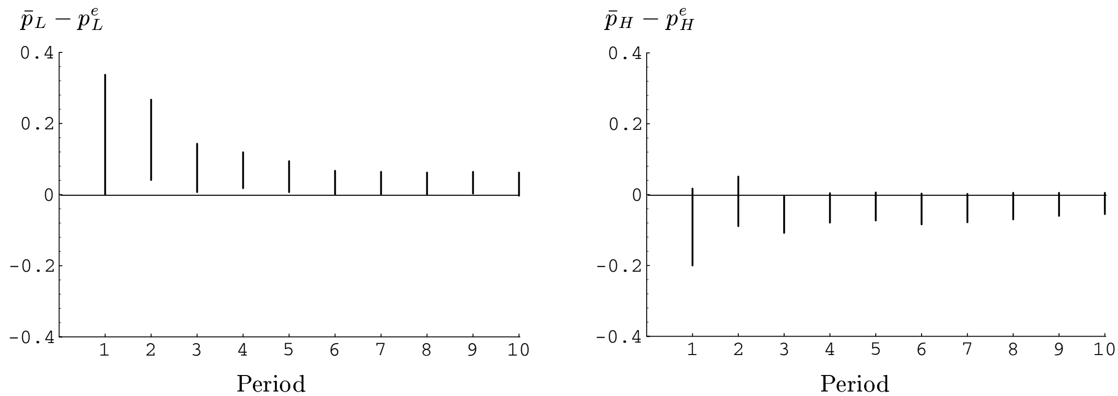


Figure 10: Confidence intervals for period mean prices in low-price markets (left) and high-price markets (right).

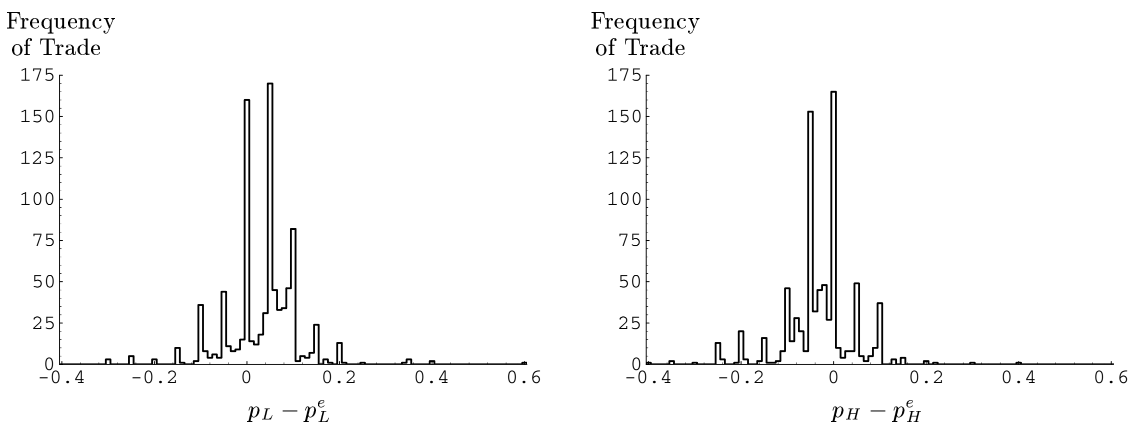


Figure 11: Histograms of deviation of transaction price from theoretical equilibrium price.

for a zero variance price series at the CE. This occurred only twice in the 170 total trading periods used in this study. (Zero variance price sequences at the equilibrium occured in market 4pda14, periods 8 and 11, commodity B; see Figure 8.) The independent variables are defined as follows:

$$t = \text{trading period time index (1-13)}$$

$$X = \begin{cases} 1, & \text{if subject experience level 2 applies} \\ 0, & \text{otherwise} \end{cases}$$

$$H = \begin{cases} 1, & \text{if the "high" CE price gap of 4.20 applies} \\ 0, & \text{otherwise} \end{cases}$$

$$L = \begin{cases} 1, & \text{if the "low" CE price gap of 2.80 applies} \\ 0, & \text{otherwise} \end{cases}$$

$$F = \begin{cases} 1, & \text{if the four-buyer market design applies} \\ 0, & \text{otherwise} \end{cases}$$

$$B = \begin{cases} 1, & \text{if market B is the high-price market} \\ 0, & \text{otherwise.} \end{cases}$$

Our a priori expectation was that only the coefficients of t, X and the constant term would be significantly different from zero (and negatively signed). In the OLS regression estimates that follow, t-values are shown in parentheses and "*" indicates significance at the 95% level of confidence.

<u>High-price Market:</u> $N = 170,\ R^2 = 0.329,\ s = 0.062$

$$\begin{array}{rcccccccc} r(t) & = & 0.182 & -\,0.012\,t & -\,0.031\,X & +\,0.003\,H & -\,0.037\,L & +\,0.002\,F & +\,0.018\,B \\ & & (12.5^*) & (-8.1^*) & (-3.1^*) & (.19) & (-3.1^*) & (.02) & (1.6) \end{array}$$

<u>Low-price Markets:</u> $N = 170,\ R^2 = 0.2671,\ s = 0.103$

$$\begin{array}{rcccccccc} r(t) & = & 0.272 & -\,0.016\,t & -\,0.052\,X & -\,0.025\,H & -\,0.049\,L & -\,0.022\,F & +\,0.009\,B \\ & & (11.2^*) & (-6.9^*) & (-3.1^*) & (-1.2) & (-2.4^*) & (-1.2) & (0.46) \end{array}$$

The subject experience (X) and time trend (t) variables are significant as expected. All other independent variables are not significant except for L. In both the high-price and low-price markets the low CE price difference tends to speed convergence to the CE relative to the other two CE price differentials. However, the high price gap (H) clearly does not retard CE convergence. We have no explanation for this empirical result other than "subject group effects" that would fade with additional market replications. To further examine the effects of the design treatments on CE convergence, we regress $r(t)$ on the X, H, L, F, and B dummy variables for the separate ($n = 15$) cross-sectional poolings corresponding to $t = 1$, 5, and 10. In all six (3 high-price, 3 low-price) OLS estimates, none of the dummy variable coefficients are significant at the 90% confidence level. We thus conclude that none of the design treatments have a consistent, robust effect on market behavior.

Individual buyer behavior The buyer parameter sets used in our experimental designs (Table 1) were initialized so that two buyers were assigned CE commodity bundles consisting of high-price and low-price units as follows: $(A_i, B_i) = (1, 3)$, $(2, 2)$, or $(3, 1)$ in the six-buyer design and $(A_i, B_i) = (2, 4)$ or $(4, 2)$ in the four-buyer design. The buyers paired at a particular CE bundle had different elasticities of substitution, either -1.33 ($\rho = 0.25$) or -4 ($\rho = 0.75$); the latter implying increased commodity substitutability and flatter indifference contours. In terms of the buyer profit tables shown in Figures 3 and 4, a higher r value translates into a reduction in the monetary opportunity cost associated with deviations from the CE bundle. For example, in the six-buyer design with $P_A^* = 3.25$ and $P_B^* = 6.75$, a buyer with $\rho = 0.75$ and a CE commodity bundle at $(A_i, B_i) = (1, 3)$ could choose to purchase only two units of the high-price commodity and spend the remaining tokens on three low-price units ending up at $(A_i, B_i) = (3, 2)$. This would result in a \$0.09 reduction in profit relative to that obtained at the CE bundle (\$1.31 vs. \$1.40). In contrast, the buyer with $\rho = 0.25$ and the same CE bundle would give up \$0.21 (\$1.19 vs. \$1.40) making the same move from $(A_i, B_i) = (1, 3)$ to $(3, 2)$. Based on the differential in the opportunity cost associated with deviations from the CE bundle, one might predict that deviations from the CE bundle will occur more frequently for buyers with $\rho = 0.75$ than for similar buyers with $\rho = 0.25$.

Table 4 presents, for each buyer classification, the percentage of end-of-period commodity bundles held by buyers that correspond to the CE prediction. The individual column labels in Table 4 specify the number of low-price and high-price units in a particular bundle rather than quantities of goods A and B since the ranking of CE prices for the two goods varied across experiments. For each $\rho = 0.75$ versus $\rho = 0.25$ CE percentage comparison, a 2×2 χ-square statistic is presented. This test statistic assumes that each trading period is an independent binomial experiment where a buyer either obtains or fails to obtain the CE commodity bundle. The null hypothesis is that the population proportion of CE outcomes under $\rho = 0.75$ will be equal to the proportion of CE outcomes under $\rho = 0.25$.

It is clear from the information in Table 4 that, while the percentage of CE outcomes is larger with $\rho = 0.25$ in 8 of the 10 comparisons using all trading periods, the difference

All trading periods

	Experience level 1					Experience level 2				
	6 buyer design			4 buyer design		6 buyer design			4 buyer design	
CE	3H,1L	2H,2L	1H,3L	4H,2L	2H,4L	3H,1L	2H,2L	1H,3L	4H,2L	2H,4L
$\rho = 0.75$	47.8%	53.6%	43.5%	9.4%	25.0%	81.8%	45.5%	6.1%	50.0%	50.0%
	(33)	(37)	(30)	(3)	(8)	(27)	(15)	(2)	(18)	(18)
$\rho = 0.25$	58.0%	56.5%	52.2%	37.5%	40.6%	48.5%	48.5%	9.1%	47.2%	55.6%
	(40)	(39)	(36)	(12)	(13)	(16)	(16)	(3)	(17)	(20)
χ^2	1.047	0.0293	0.726	5.573*	1.134	6.673*	0	0	0	0.056
Trading periods 6 – end only										
$\rho = 0.75$	61.5%	66.7%	51.3%	11.8%	35.3%	88.9%	50.0%	5.6%	47.6%	42.9%
	(24)	(26)	(20)	(2)	(6)	(16)	(9)	(1)	(10)	(9)
$\rho = 0.25$	76.9%	61.5%	69.2%	52.9%	52.9%	61.1%	44.4%	16.7%	47.6%	52.4%
	(30)	(24)	(27)	(9)	(9)	(11)	(8)	(3)	(10)	(11)
χ^2	1.505	0.056	1.927	4.838*	0.4772	2.37	0	0.281	0	0.095

*Significant at $p \leq 0.05$. (Direction not predicted.)

Table 4: Percentage of buyer quantity choices at CE.

is statistically significant (in the predicted direction) only for the $(4H, 2L)$ pairing in the four-buyer design under experience level 1. However, a matched-pairs Wilcoxon test using the ten paired observations in the upper part of Table 4 yields $Z = 1.682$ (barely large enough to reject the null hypothesis of identical populations at the $p = .05$ level, direction predicted). It appears that the opportunity cost differential generated by the variation in ρ is simply too small to induce the predicted differences in behavioral outcomes with very much consistency. Pooling observations across buyer CE classes and experience levels, we find that 43.2% of the buyer quantity outcomes (191 of 442) are at the CE when $\rho = 0.75$ and 48% are at the CE when $\rho = 0.25$ (212 of 442). The difference is not statistically significant at the .05 level ($\chi^2 = 1.82$, $p = 0.177$).

From Table 4 we can also note that the frequency of CE outcomes is not consistently larger in the experience level 2 experiments relative to experience level 1. Pooling across designs and buyer classifications we find that under level 1 subject experience 46.3% of the buyer quantity outcomes are at the CE (252 of 542) compared with 44.4% under level 2 (151 of 341). The difference is not significant ($\chi^2 = 0.27$, $p = 0.6$).

Additional perspective on buyer performance relative to the CE prediction is gained from looking at the actual earnings of buyers relative to the CE prediction. Over all trading periods, buyers realized an average of 87.33% of the profit available at the CE. Deleting observations from the first five periods, the average earnings rises to 90.84% of CE earnings. All buyers obtained 100% of the CE profit in only 8.2% (14 of 170) trading periods. While deviations from the CE were obviously very common, buyers were generally "close" to the CE commodity bundle; frequently one high-price or low-price unit away. It is important to realize that buyers did not face a fixed price (at the CE or anywhere else) while making their purchase decisions. Given the uncertainty embodied in the problem they faced, deviations from the CE commodity bundle should not be interpreted as constituting deviations from "rational", utility-maximizing behavior.

Summary of experimental results The two-commodity double auctions reported in this study tend to converge toward a behavioral price-quantity equilibrium that is near the competitive theoretic equilibrium. After about three to five trading periods, the mean price tends to stabilize slightly above the competitive prediction in the "low-price" market and slightly below the competitive prediction in the "high-price" market (where in our design $P_{hi}/P_{lo} = 2.077$ and $P_{hi} - P_{lo} = 2.80$, 3.50, or 4.20). Trading volume is slightly below the CE prediction on average, with the low-price market displaying somewhat higher volume than the high-price market. Around 45% of all end-of-period commodity bundles held by buyers coincide with the prediction from the competitive model. A smaller proportion of choices at the competitive equilibrium bundle is observed for buyers with an elasticity of commodity substitution of -4 (43.2%) versus -1.33 (48%). This result is consistent with the fact that the opportunity cost of deviating from the competitive equilibrium bundle is smaller for buyers with the higher elasticity of substitution (flatter indifference contours). The difference, however, is not statistically significant nor is it robust with respect to variations in buyer groupings. While deviations from the CE commodity bundle are quite common, buyers manage to earn an average of over 87% of the profits available at the CE.

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